

Translation: Attempt of a Theory of Electrical and Optical Phenomena in Moving Bodies/Section V

1 Application to optical phenomena.

1.1 Reduction to a resting system.

§ 56. The specification of the influence, that the motion of ponderable bodies exerts on the phenomena of light, can be achieved in a very simple manner, if we neglect circular polarization, as it will always take place in this section.

Namely we want, as we did it earlier (§ 31) already, by continuing omission of magnitudes of second order, to introduce (instead of t) the “local time”

$$t' = t - \frac{1}{V^2} (\mathfrak{p}_x x + \mathfrak{p}_y y + \mathfrak{p}_z z)$$

as an independent variable; besides we want (instead of \mathfrak{D}) consider a new vector \mathfrak{D}' , which we define by the formula

If we consider an arbitrary magnitude as a function of x , y , z and t' , then (as before (§81)) we denote the partial derivative by

$$\left(\frac{\partial}{\partial x}\right)', \left(\frac{\partial}{\partial y}\right)', \left(\frac{\partial}{\partial z}\right)', \frac{\partial}{\partial t'}.$$

Furthermore, according to this notation, we shall understand by

$$Div' \mathfrak{A}$$

the expression

$$\left(\frac{\partial \mathfrak{A}_x}{\partial x}\right)' + \left(\frac{\partial \mathfrak{A}_y}{\partial y}\right)' + \left(\frac{\partial \mathfrak{A}_z}{\partial z}\right)',$$

and by

$$Rot' \mathfrak{A}$$

a vector with the components

$$\left(\frac{\partial \mathfrak{A}_z}{\partial y}\right)' - \left(\frac{\partial \mathfrak{A}_y}{\partial z}\right)' \text{ etc.}$$

The introduction of t' and \mathfrak{D}' gives the advantage, that (as I will show now) the equations (I_c) — (V_c) assume the same form as the formulas that apply to $\mathfrak{p} = 0$.

§ 57. At first we obtain, by consideration of formulas (35),

$$Div \mathfrak{D} = Div' \mathfrak{D} - \frac{1}{V^2} (\mathfrak{p}_x \dot{\mathfrak{D}}_x + \mathfrak{p}_y \dot{\mathfrak{D}}_y + \mathfrak{p}_z \dot{\mathfrak{D}}_z),$$

or by (III_c), if we replace (in the terms multiplied by $\mathfrak{p}_x, \mathfrak{p}_y, \mathfrak{p}_z$) \mathfrak{H}' by \mathfrak{H} and Div by Div'

Hence the equation (I_c) becomes

In a similar way

$$Div \mathfrak{H} = Div' \mathfrak{H} - \frac{1}{V^2} (\mathfrak{p}_x \dot{\mathfrak{H}}_x + \mathfrak{p}_y \dot{\mathfrak{H}}_y + \mathfrak{p}_z \dot{\mathfrak{H}}_z)$$

i.e., by (IV_c),

so that it can be written for (II_c)

Now let us turn to formula (III_c). In this one, three equations are summarized, namely in the first of them on the left side, the expression

$$\frac{\partial \mathfrak{H}'_z}{\partial y} - \frac{\partial \mathfrak{H}'_y}{\partial z}.$$

is stated. For that, we can write with respect to (35)

$$[Rot' \mathfrak{H}]_x - \frac{1}{V^2} \left\{ \mathfrak{p}_y \frac{\partial \mathfrak{H}'_z}{\partial t'} - \mathfrak{p}_z \frac{\partial \mathfrak{H}'_y}{\partial t'} \right\},$$

and thus for the equation itself

$$[Rot' \mathfrak{H}]_x = 4\pi \frac{\partial \mathfrak{D}_x}{\partial t'} + \frac{1}{V^2} \frac{\partial}{\partial t'} \{ \mathfrak{p}_y \mathfrak{H}_z - \mathfrak{p}_z \mathfrak{H}_y \} = 4\pi \frac{\partial \mathfrak{D}_x}{\partial t'}$$

The two other equations admit of a similar transformation, and therefore we have

Furthermore, as regards the first of equations (IV_c), this one goes over, since

$$\frac{\partial \mathfrak{E}_z}{\partial y} - \frac{\partial \mathfrak{E}_y}{\partial z} = [Rot' \mathfrak{E}]_x - \frac{1}{V^2} \left\{ \mathfrak{p}_y \frac{\partial \mathfrak{E}_z}{\partial t'} - \mathfrak{p}_z \frac{\partial \mathfrak{E}_y}{\partial t'} \right\}$$

into

$$[Rot' \mathfrak{E}]_x = -\frac{\partial \mathfrak{H}_x}{\partial t'} + \frac{1}{V^2} \frac{\partial}{\partial t'} \{ \mathfrak{p}_y \mathfrak{E}_z + \mathfrak{p}_z \mathfrak{E}_y \} = -\frac{\partial \mathfrak{H}'_x}{\partial t'}$$

so that (IV_c) is equivalent with

Eventually it follows from

§ 58. To introduce the new variables also into the *limiting conditions*, we consider the perpendicular n for the considered point, and also two directions h and k that are perpendicular to one another and to n . There, the direction n shall correspond to a rotation by a right angle from h to k . Consequently it follows from (IX) (§ 56)

Now, since \mathfrak{D}_n , \mathfrak{H}'_k and \mathfrak{H}'_h are steady, then this must also be so for \mathfrak{D}'_n .

In a similar manner we derive from the continuity of \mathfrak{H}_n , \mathfrak{E}_h and \mathfrak{E}_k , by means of the relation to be derived from (VI_c)

$$\mathfrak{H}'_n = \mathfrak{H}_n - \frac{1}{V^2} [\mathfrak{p} \cdot \mathfrak{E}]_n = \mathfrak{H}_n - \frac{1}{V^2} [\mathfrak{p}_h \mathfrak{E}_k - \mathfrak{p}_k \mathfrak{E}_h],$$

the continuity of \mathfrak{H}'_n .

If we also notice the other equations ($VIII_c$), then it is clear, that all limiting conditions are contained in the formulas

in which h can be now any arbitrary direction in the border surface.

§ 59. The equations (I_d) – (V_d) and ($VIII_d$) differ from the equations which apply to stationary bodies by § 52, only by the fact that

$$t', \mathfrak{D}' \text{ und } \mathfrak{H}'$$

has taken the place of

$$t, \mathfrak{D} \text{ und } \mathfrak{H}$$

This coincidence opens for as a way, to treat problems regarding the influence of Earth's motion on optical phenomena, in a very simple way.

Namely, if a state of motion for a system of stationary bodies is known, where

are certain functions of x, y, z and t , then in the same system, if it is displaced by the velocity \mathfrak{p} , there can exist a state of motion, where

are exactly the same functions of x, y, z and t' [that is, $t - \frac{1}{V^2} (\mathfrak{p}_x x + \mathfrak{p}_y y + \mathfrak{p}_z z)$].

Although we have given (in the previous consideration) to the coordinate axes the directions of the symmetry axis, the derived theorem applies to any right-angled coordinate system. We can easily recognize this, when we consider, that for local time t' it can also be written

$$t - \frac{\mathfrak{p}_r r}{V^2},$$

where r is the line drawn from the coordinate origin to the point (x, y, z) , and t' is independent of the *direction* of the coordinate axes.

We may remember the fact, that in a moving system we always have to understand by x, y, z the coordinates with respect to the axes that share the translation.

If the magnitudes (70) are known as functions of x, y, z and t' , thus also as functions of x, y, z and t , then $\mathfrak{D}_x, \mathfrak{D}_y, \mathfrak{D}_z, \mathfrak{H}_x, \mathfrak{H}_y, \mathfrak{H}_z$ can be calculated from the equations (IX) and (VI_c).

1.2 Different applications.

§ 60. We want to call the two states of motion — in the stationary and in the moving system of bodies —, of which we have spoken so far, *corresponding* states. Now, they shall be mutually compared more precisely.

a. If in a stationary system the magnitudes (69) are periodic functions of t with the period T , then in the other system the magnitudes (70) have the same period with respect to t' , thus also with respect to t , when we let x, y, z remain constant. When interpreting this result, we have to consider, that two periods must be distinguished in the case of translation (see §§ 37 and 38), which we accordingly can call *absolute* and *relative* period. We are dealing with the absolute one, when we consider the temporal variations in a point that has a fixed position against the aether; but we are dealing with the relative one, when we consider a point that moves together with ponderable matter. The things found above can now be expressed as follows:

If a state of oscillation in the moving system shall correspond to a state in the stationary system, then the relative oscillation period in the first mentioned case must be equal to the oscillation period in the second mentioned case.

b. In the stationary system, no motion of light may take place at an arbitrary location, i.e., $\mathfrak{D}, \mathfrak{E}$ and \mathfrak{H} may vanish at this place. At the corresponding location of the moving bodies it is consequently $\mathfrak{D}' = 0, \mathfrak{E} = 0, \mathfrak{H}' = 0$, thus also $\mathfrak{D} = 0, \mathfrak{H} = 0$, so that at this place the motion of light is missing as well.

From that it directly follows, that a surface that forms the border of a space filled with light within a stationary body, can have the same meaning when the body is moving.

In a stationary, homogeneous medium, for example, light bundles are possible which are limited by cylindric surfaces, if it is only assumed that the dimensions of the bi-sections are much greater than the wave length. *By our theorem, such bundles also can exist in a moving system.*

The described lines of the mentioned cylindric surfaces we call *light rays*, and in the case of translation: *relative* light rays. The cylinders we have to imagine as rigidly connected with ponderable matter; thus they form the paths for the propagation of light relative to that matter.

c. A cylindric light bundle falls upon a plane limiting-surface in a stationary system, and it will be mirrored and refracted by it, — for generality we want to say: bi-refracted. The new light bundles have a cylindric border as well. If we now apply the things said under *a* and *b* to the corresponding case of the moving system, then we come to the theorem:

In the moving system, relative light rays of relative oscillation period T were mirrored and refracted by the same laws, as rays of the oscillations period T in the stationary system.

d. Let in the stationary system be a transparent body of arbitrary form, that was hit by a cylindric light bundle, and by that an arbitrary interference- or diffraction-phenomenon occurs. If dark strips do occur on that occasion, then they must appear in the corresponding state of the moving system at exactly the same locations.

An extreme case of a diffraction-phenomenon is the unification of all light in a *focus*. By the preceding, the laws by which a light ray of certain cylindric limitation is concentrated by a telescope objective, won't be changed at all by a translation.

e. While in corresponding states the *lateral limitation* of a light ray is the same, the *wave normals* have different directions. If it is set, for example, that plane waves are propagating with the velocity W in the stationary system whose perpendicular has the direction (b_x, b_y, b_z) , so that the deviation from equilibrium is a function of

$$t - \frac{b_x x + b_y y + b_z z}{W}$$

then for the moving system, similar functions of

$$t' - \frac{b_x x + b_y y + b_z z}{W} = t - \left\{ \left(\frac{b_x}{W} + \frac{p_x}{V^2} \right) x + \left(\frac{b_y}{W} + \frac{p_y}{V^2} \right) y + \left(\frac{b_z}{W} + \frac{p_z}{V^2} \right) z \right\}$$

occur. The direction constants b'_x, b'_y, b'_z of the wave normal will thus be determined for this system by the condition

$$b'_x : b'_y : b'_z = \left(b_x + \frac{W p_x}{V^2} \right) : \left(b_y + \frac{W p_y}{V^2} \right) : \left(b_z + \frac{W p_z}{V^2} \right),$$

or, in the case of a propagation in pure aether, by

$$b'_x : b'_y : b'_z = \left(b_x + \frac{p_x}{V} \right) : \left(b_y + \frac{p_y}{V} \right) : \left(b_z + \frac{p_z}{V} \right).$$

From this equation is given in reverse

1.3 The aberration of light.

§ 61. Let b'_x, b'_y, b'_z be the direction constants of the line drawn from a stationary celestial body to earth, thus also

the direction constants of the perpendicular with respect to the plane waves that arrive in the vicinity of earth. So when we, to investigate the following path of propagation, relate the motion of light to a coordinate system, that shares the motion of earth, then of course the direction constants of the wave normal remain b'_x, b'_y, b'_z , while that one comes into play as the relative oscillation period T' (§ 37), which was modified by Doppler's law. As we have seen, the motion (as regards the lateral limitation of a light bundle cut out by a diaphragm, the concentration through lenses, and the passage through other transparent bodies) will correspond to a motion in a stationary system, for which the oscillation period is T' , and the perpendicular to the incident waves has the direction constants b_x, b_y, b_z that are to be determined by (71).

Thus all phenomena happen exactly in such a manner, as if the earth were at rest, the oscillation period were T' , and the celestial body, as seen from earth, would be located not in the direction $(-b'_x, -b'_y, -b'_z)$, but in the direction $(-b_x, -b_y, -b_z)$.

Now, *aberration* exactly consists of the latter. That the magnitude and direction, which we find for it, actually corresponds to the known rule which is in accordance with observations, follows immediately from equation (71). Namely, we obtain a vector of direction (b_x, b_y, b_z) , when we compose a vector of direction (b'_x, b'_y, b'_z) , whose length represents the velocity of light, with a second one which is equal and opposite to Earth's velocity p .

By the way, in our theorem also lies the explanation for the fact, that during the observation by a lens system, always that aberration arises which is determined by the previously mentioned rule^[1], as well as the explanation for the known experiments of Arago^[2] by a prism, and for the experiment proposed by Boscovich and executed by Airy, in which the tube of a telescope was filled with water^[3].

1.4 Observations by sun light.

§ 62. The trajectory of Earth deviates as little from a circle, that, when we are dealing with sun rays, we can neglect the velocity component p_r , on which the variation of the oscillation period depends (§ 37). Experiments with these rays must have the result, as if the earth were at rest, and the sun were in a direction changed by aberration, and *would emanate types of light of the same oscillation period, as in reality*^[4]

From that it immediately follows, that (as regards a certain line of Fraunhofer during a refraction in a prism, or the diffraction through a lattice) we *don't* register any influence of Earth's motion, thus it cannot make any difference, whether the direction of light (that falls upon the prism of the lattices) would form this or that angle with the translation of earth. As regards the lattice-spectra, this result was confirmed by the careful experiments of

Mascart^[5]. This physicist has additionally demonstrated by certain experiments^[6], that as regards the mentioned spectra, the deflection for a certain Fraunhofer line fully agrees with the deflection for the corresponding rays of a terrestrial light source^[7].

1.5 Moving light sources.

§ 63. Above, in § 61, the celestial body was assumed to be at rest. Yet also for a moving body we arrive at a simple result. We already know (§ 36), that the perpendicular to the waves arriving at Earth, is directed to location P , where the light source was present in the instant when the light was emitted. Now the motion of Earth causes, that we observe the star not at this place P , but at another place P' , namely the displacement from P to P' can be derived by the ordinary rule for aberration. By the consideration of § 61 its prove is at hand.

Eventually a simple figure shows, *that P' falls into the true place at the time of observation, as soon as the velocity of the light source agrees in magnitude and direction with that of earth.*

1.6 Experiments with terrestrial light sources.

§ 64. From the results previously obtained it directly follows, that we will see a distant terrestrial object always in the direction, where it is actually located. We also have already seen, that for a light sources rigidly connected with earth, no difference exists between the true and the observed oscillation period.

In general, the motion of Earth will never have an influence of first order on the experiments with terrestrial light sources.

To justify this theorem, we want at first (by application of the superposition principle (§ 7)) derive from the formulas of § 33 other ones, which are valid for an arbitrary system of luminous molecules. On that occasion we assume, that they have the common translation \mathfrak{p} , and we choose the local time t' specified by (34), and the relative coordinates (§ 19), as independent variables.

Let

$$(\xi_1, \eta_1, \zeta_1), (\xi_2, \eta_2, \zeta_2), \text{ etc.}$$

be the locations of the molecules, and

or

be the electric moments that occur within.

The condition that was caused by a single molecule in the point (x, y, z) of the aether, will be determined by equations (39) and (40). The latter one we additionally want to transform (to subsequently apply the theorem of § 59

more conveniently) by introducing the expressions \mathfrak{D} and \mathfrak{D}' for the aether. For this medium, as we know, \mathfrak{D} is equal to \mathfrak{d} , and thus by (IX) (§ 56), $4\pi V^2 \mathfrak{D}'$ is equal to

$$4\pi V^2 \mathfrak{d} + [\mathfrak{p} \cdot \mathfrak{s}].$$

By means of equation V_b) we may replace \mathfrak{F} by $4\pi V^2 \mathfrak{D}'$ in (40).

Furthermore, if we denote by Σ the sum of terms, any of them stemming from a luminous molecule, then we obtain from (39) and (40) the following formulas for the condition in the aether caused by ion oscillations (72):

Here, r denotes the distance of point (x, y, z) from the location (ξ, η, ζ) of one of the luminous molecules, while $\mathfrak{m}_x, \mathfrak{m}_y, \mathfrak{m}_z$ represent the moments of this molecule at local time $t' - \frac{r}{V}$. The two first members of the sum

$$\sum \left(\frac{\mathfrak{m}_x}{r} \right)$$

are for example

$$\frac{1}{r_1} f_1 \left(t' - \frac{r_1}{V} \right) \text{ und } \frac{1}{r_2} f_2 \left(t' - \frac{r_2}{V} \right),$$

when r_1 and r_2 are the distances between (x, y, z) and the two first molecules.

§ 65. From the preceding formulas, others immediately arise, which apply to a *stationary* light source when we simply erase all accents. If in this case in the luminous molecules the moments exist

then we have in the aether

where $\mathfrak{m}_x, \mathfrak{m}_y, \mathfrak{m}_z$ are now the moments of a molecule at time $t - \frac{r}{V}$, so that *e.g.* the two first members of the sum

$$\sum \left(\frac{\mathfrak{m}_x}{r} \right)$$

have the values

$$\frac{1}{r_1} f_1 \left(t - \frac{r_1}{V} \right) \text{ und } \frac{1}{r_2} f_2 \left(t - \frac{r_2}{V} \right)$$

Of course, $\xi, \eta, \zeta, x, y, z$ are now the coordinates related to *stationary* axes.

§ 66. The two cases considered in §§ 64 and 65 (*with* or *without* translation) shall be compared to one another. Here, we imagine that the spatial arrangement of the luminous molecules is the same in the two cases, *i.e.* that all ξ, η, ζ have the same value; the latter we also assume for x, y, z , with the result that we consider the state of the aether in a point that has a particular location with respect to the light source. Eventually we understand by f_1, g_1, h_1, f_2 etc. the same function-sign for both cases.

A look upon the formulas (74) and (76) let us recognize, that we are dealing with *corresponding* states, on which the theorem of § 59 is applicable. If the light is incident on a non-transparent screen with one opening, then

the limitation of light and shadow, or the location of dark diffraction fringes behind of it, will be the same in both cases. Also no difference in the spatial distribution of light and dark will be seen, when the rays were mirrored or refracted at an arbitrary transparent body, or when a lens concentrates them, or when some interference phenomena occur.

Of course, motions that are present in the light source itself, which generate these corresponding states, are not quite the same. In one case they will be determined by (73), and in the other case by (75). If we put

$$f_1 \left(t - \frac{p_x}{V^2} \xi_1 - \frac{p_y}{V^2} \eta_1 - \frac{p_z}{V^2} \zeta_1 \right) = f'_1(t), \text{ etc.,}$$

then we may thus also say:

A moving light source, in which ion motions as represented by

take place, generates the same phenomena as a stationary light source, to which the formulas

apply.

If we are dealing with oscillations, then the difference between (77) and (78) is reduced to a *variation of the phases*, namely this will be determined for an arbitrary molecule by

$$\frac{p_x}{V^2} \xi + \frac{p_y}{V^2} \eta + \frac{p_z}{V^2} \zeta$$

consequently it is *not equal for the various molecules*.

It is now to be noticed, that the molecules of a light source, *e.g.* a flame, must be considered as totally independent from one another, so that, as it is ordinarily expressed, the rays emanated by two of these particles cannot mutually interfere. From that it follows, that arbitrary variations in the phases of the single molecules cannot have any influence on the observable phenomena. The stationary light source with motions (78) will give nothing other than a stationary source (also at rest) with motions (77), and thus we may claim:

If we set a light source into translation, without changing anything of the oscillations of their ions, then the observable phenomena in bodies rigidly connected with them, remain as they were.

§ 67. Numerous experiments have proven, that when using terrestrial light sources, the phenomena are indeed independent of the orientation of the devices with respect to the direction of Earth's motion. Here, the observations of Respighi,^[8] Hoek,^[9] Ketteler^[10] and Mascart^[11] on refraction do belong, as well as the experiments of the three last mentioned physicists on interference phenomena.^[12] We are indebted to Ketteler for an investigation on the inner reflection and the refraction at calcite prisms.^[13] and to Mascart for an investigation^[14] on the interference fringes that appear at calcite plates in polarized light.

1.7 The entrainment of light waves by ponderable matter.

§ 68. In a stationary, isotropic or anisotropic body a bundle of plane light waves propagate, as to which the components of \mathfrak{D} and \mathfrak{H} can be expressed by expressions of the form

thus W is the velocity of propagation. This magnitude can depend on b_x, b_y, b_z , and T . After we have given to the body the velocity \mathfrak{p} , a state of motion can occur in it, for which expressions like

$$A \cos \frac{2\pi}{T} \left(t' - \frac{b_x x + b_y y + b_z z}{W} + B \right)$$

or

apply. The direction constants b'_x, b'_y, b'_z of the wave normal are now proportional to the magnitudes

$$\frac{b_x}{W} + \frac{p_x}{V^2}, \frac{b_y}{W} + \frac{p_y}{V^2}, \frac{b_z}{W} + \frac{p_z}{V^2}$$

If we consequently put

then (80) becomes

$$A \cos \frac{2\pi}{T} \left\{ t - \frac{b'_x x + b'_y y + b'_z z}{W'} + B \right\}$$

for which we can see, that W' is the velocity by which the waves of *relative* oscillation period T are propagating in the direction (b'_x, b'_y, b'_z) within the moving body.

From (81) we find

$$\frac{1}{W'^2} = \frac{1}{W^2} + 2 \frac{b_x p_x + b_y p_y + b_z p_z}{W V^2},$$

and for that we can write, neglecting magnitudes of second order,

$$\frac{1}{W'^2} = \frac{1}{W^2} + 2 \frac{b'_x p_x + b'_y p_y + b'_z p_z}{W V^2} = \frac{1}{W^2} + 2 \frac{p_n}{W V^2}.$$

Here, p_n is the component of velocity into the direction of the wave normal, with which W' is related. Eventually

§ 69. Up to now, the investigation was general. Now it shall be assumed, that the body be isotropic. The velocity W is thus independent of the direction of the waves, and also the ratio

$$\frac{V}{W} = N,$$

the absolute refraction index of the stationary body, only depends on T .

When interpreting formula (82), which now passes to

we have to remember, that we have used a coordinate system for the description of the phenomena, that moves

together with ponderable matter. Thus (83) is the velocity of the light waves *relative to that matter*. If we wish it know the relative velocity W'' *with respect to the aether*, we have to compose the velocity (83), which has the direction of the wave normal, with the component p_n of the translation velocity (that exactly falls in that direction). By that we obtain

which is in agreement with the known assumption of Fresnel.

As regards this result, two things shall be remarked. *First*, the given derivation applies to every value of T , thus *for every kind of light*, and *second*, this has to be understood, that the substitution of the values of N and W , which belong in the stationary body to a particular T , gives the value of W'' for the relative oscillation period T .^[15]

§ 70. If the considered body is birefringent, than it may not be forgotten, that W and W' in equation (82) are related to *different* directions of the wave normal, namely W to direction (b_x, b_y, b_z) , and W' to direction (b'_x, b'_y, b'_z) . Concerning the question, as to how the velocities in stationary and in moving bodies are mutually different *for a given direction of the waves*, the equation doesn't directly give an answer. To a simple theorem, however, leads the introduction of *light rays*.

In a stationary birefringent body, to any direction of the wave normal (as soon as one of the two possible oscillation directions is chosen) belongs a particular direction for the light-rays, *i.e.*, for the describing lines of a cylindric limiting-surface of a light bundle. For the points of such a line, it is now, when c_x, c_y, c_z are the direction constants, and s means the distance of a fixed point (x_0, y_0, z_0) of the line,

By that, when we put

$$\frac{W}{b_x c_x + b_y c_y + b_z c_z} = U$$

and understand by B' a new constant, the expression (79) is transformed into

$$A \cos \frac{2\pi}{T} \left(t - \frac{s}{U} + B' \right).$$

The magnitude U is, what we usually call the *velocity of the light ray*.

If we now pass to the corresponding motion in the progressing body, then the considered line *remains* (§ 60, *b*) a light ray, and we obtain for the determination of the deviations of equilibrium, in the different points of it, expressions as

$$A \cos \frac{2\pi}{T} \left(t' - \frac{s}{U'} + B' \right).$$

or, by (34) and (85),

where p_s is the component of p in the direction of the light ray, while the new constant B'' has the value

$$B' - \frac{p_x x_0 + p_y y_0 + p_z z_0}{V^2}$$

The expression (86) goes over into

$$A \cos \frac{2\pi}{T} \left(t' - \frac{s}{U'} + B'' \right).$$

and here, U' is the velocity of the light ray in the moving body, when we put

$$\frac{1}{U'} = \frac{1}{U} + \frac{p_s}{V^2}$$

From that we conclude

a formula, whose shape agrees with (82), in which U and U' are now related to *light rays of the same direction*.

§ 71. Formula (84) has found a nice confirmation by the experiment, that were first executed by Fizeau and later repeated by Michelson and Morley^[16], on the propagation of light in streaming water. The arrangement of them should be sufficiently known, so that we can restrict ourselves to compare (still more deeply than it is usually happening) the results with the theory.

To apply the formula, we first have to derive the relative period from the experimental conditions, and then (from the dispersion formula for stationary water) the refraction exponent N corresponding with this period. The value of V/N calculated in this way, we eventually have to substitute into (82) for W . However, as regards the relative period, a more closer consideration is required.

It's known that, as regards these experiment, two tubes were used which are closed by glass plates and which are lying next to one another, through which the water was flowing with the same velocity, but in different direction; since the base tubes were present entirely at the edges, we may assume, that at all places (at least in the middle parts of the bisection) the same velocity p occurred^[17]. The two light bundles, which should mutually interfere, passed through the device, so that one was propagated in *both* tubes in the direction of the water stream, and the other one steadily in the opposite direction.

We now consider a fixed point P in the interior of one of the tubes. The conditions, under which the light is propagating from the source to this point, obviously remain — when the water stream is stationary — constantly the same, and namely this applies to *both* ways, on which the rays can reach point P . Impulses, which emanate by certain periods from the source, will arrive with the same periods in P , and when T is the oscillation period of the light source, then this is also the absolute oscillation period in P .

From that it follows for the *relative* oscillation period related to the water

where W' is exactly the sought velocity of the waves, while (as also in the following formulas) the above or below sign is to be applied, depending on whether the considered light bundle is propagating in the direction of the water motion, or in the opposite direction.

We always neglect magnitudes of second order and thus we may put instead of (88)

Under W in equation (82) — and also in this expression (89) itself — we have now to understand the value, which belong to period (89) in the stationary body. The corresponding refraction exponent is

$$n \pm \frac{p}{W} T \frac{dn}{dT},$$

in case we denote the refraction exponent for period T by n ; consequently we have to substitute

$$W = \frac{V}{n \pm \frac{p}{W} T \frac{dn}{dT}} = \frac{V}{n} \mp \frac{p}{n^2} \frac{V}{W} T \frac{dn}{dT},$$

or, when we replace W by $\frac{V}{n}$ in the last member,

$$W = \frac{V}{n} \mp \frac{p}{n} T \frac{dn}{dT}.$$

Furthermore it is in (82)

$$p_n = \pm p,$$

so that we find

$$W' = \frac{V}{n} \mp \frac{p}{n^2} \mp \frac{p}{n} T \frac{dn}{dT},$$

and for the relative velocity with respect to the aether, thus also with respect to the closing plates of the tubes,

§ 72. The mentioned physicists have compared their observations, not with that formula, but with another in which the last term is missing; a satisfying agreement occurred at this place. Namely, if we put

$$W'' = \frac{V}{n} \pm p\epsilon,$$

thus the coefficient ϵ can be derived from the experiments. Now, while Michelson and Morley found in this manner

$$\epsilon = 0,434,$$

“with a possible error of $\pm 0,02$ ”, $1 - \frac{1}{n^2}$ for D -light has the value 0,438.

By our theory it should be

$$\epsilon = 1 - \frac{1}{n^2} - \frac{1}{n} T \frac{dn}{dT}$$

or, if we consider n as a function of wave length λ in air,

$$\epsilon = 1 - \frac{1}{n^2} - \frac{1}{n} \lambda \frac{dn}{d\lambda}$$

For the Fraunhofer line D , this becomes

$$0,451.$$

Thus formula (90) somewhat further deviates from the observations than the simpler equation

however, the observation were possibly not as exact for allowing us to put weight to this condition.

If it should be achieved, which namely appears to be difficult but not impossible, to experimentally distinguish between the equations (90) and (91), and if the first one should be justified, then we would have observed the Doppler variation of the oscillation period for an artificially generated velocity. It is only by consideration of this variation, that we have derived equation (90).

§ 73. It is hardly necessary to recall at this place the importance of the role, which is played by formula (84) in the theory of aberration and the related phenomena. Fresnel based his explanation of Arago's prism experiment upon the value $1 - \frac{1}{N^2}$ of the dragging coefficient. Subsequent scientists have applied this equations to many other cases, and have derived from it, that the motion of earth, as regards most of experiments with terrestrial light sources, is without influence, and that experiments with the light of a celestial body must give a result, as if the direction altered by aberration would be the real one. How easy the theoretical considerations are formed, when we look, not upon the direction of the waves, but on the *path of light rays*, I have demonstrated (following the example of Veltmann^[18]) in my treatise of the year 1887.^[19] At that time, I restricted myself to isotropic bodies, since it wasn't known to me yet, how to extend Fresnel's law for crystals. Now, since it was demonstrated, that the propagation velocity of light rays obeys in these bodies the simple law expressed in formula (87), it is easy to show, *that also the birefringence of rays is independent of Earth's motion.*^[20] For this purpose we can start with a simple theorem that follows from the principle of Huygens, and I allow myself to shortly state it at this place.

Let A and B two arbitrary, which may lie within different mutually adjacent media. In general, only a restricted amount of light rays can travel from one to another. If we now form (for one such ray, as well as for other ways between A and B with only small deviations) the integral

$$\int \frac{ds}{U},$$

in which U means the velocity for a light rays that follows the line element ds , then (by the referred theorem) the integral for the light ray is a minimum.

However, I neither want to dwell more closely on these considerations, nor on further applications of formulas (82) and (87), since we have solved the question concerning the influence of Earth's motion in different cases, already above in a much simpler way.

1.8 Closer consideration of light bundles of plane waves.

§ 74. In the application of the general theorem found in § 59, I was as brief as possible and I didn't dwell more into the details, as it just was required. For further explanation is seems justified, however, to show by some examples, as to how all details of the light motions are given from that theorem as well.

At first, we consider a light bundle of plane waves, that propagates in the aether, after it went through an additional opening in a non-transparent screen which is connected to Earth. For a moment we neglect the motion of Earth. Let:

l, m, n the direction constants of the wave normal,

q a constant,

f, g, h the direction constants of the dielectric displacement,

a the "amplitude" of the latter.

Consequently, the light motion can be represented by the equations

with the condition

We can easily see, that these values satisfy all equations of motion. The vectors \mathfrak{d} and \mathfrak{h} are perpendicular to one another and to the wave normal; the direction of the light rays (§ 60, b) falls into the latter.

§ 75. If the Earth is moving, then by the theorem of § 59 a condition is possible, which (related to a moving coordinate system), will be represented by

By \mathfrak{d}' we have to understand a vector \mathfrak{D}' for the pure aether, which is defined by (IX) (§ 56).

While the light rays, which determine the lateral limitation of the bundle, have still the direction (l, m, n), the wave normal deviates from it. Its direction constants l', m', n' satisfy, as it can be seen from (98), the conditions

$$l' : m' : n' = \left(l + \frac{p_x}{V}\right) : \left(m + \frac{p_y}{V}\right) : \left(n + \frac{p_z}{V}\right).$$

We will neglect all magnitudes of second order again. Then, by denoting the components of \mathfrak{p} in the direction of the rays by p_s , we have

by which (98) is transformed into

$$\psi' = \frac{2\pi}{T} \left\{ t - \left(1 + \frac{p_s}{V}\right) \frac{l'x + m'y - n'z}{V} + q \right\}.$$

While T is now the *relative* oscillation period, we find for the *absolute* one (§§ 60 and 37)

$$T' = T \left(1 - \frac{p_s}{V}\right).$$

For the determination of \mathfrak{d} and \mathfrak{h} , the formulas (IX) (§ 56) and ($V I_b$) (§ 20) can serve, which we may replace by

$$4\pi V^2 \mathfrak{d} = 4\pi V^2 \mathfrak{d}' - [\mathfrak{p} \cdot \mathfrak{h}']$$

and

$$\mathfrak{h} = \mathfrak{h}' + 4\pi [\mathfrak{p} \cdot \mathfrak{d}']$$

If follows

or, if we put by (99)

$$\frac{p_x}{V} = l' \left(1 + \frac{p_s}{V}\right) - l, \text{ etc.}$$

and if we consider (95).

$$\mathfrak{h}_x = 4\pi a V \left(1 + \frac{p_s}{V}\right) (m'h - n'g) \cos \psi', \text{ etc.}$$

By that we see, that \mathfrak{d} and \mathfrak{h} are both perpendicular to the wave normal, as it was expected. Additionally, both vectors are mutually perpendicular, which can be seen most easily, when by replace (100) by

$$\mathfrak{d}_x = a \left\{ f - \frac{p_y}{V} (l'g - m'f) + \frac{p_z}{V} (n'f - l'h) \right\} \cos \psi', \text{ etc.}$$

Furthermore, we can conclude, that the vector $[\mathfrak{d} \cdot \mathfrak{h}]$ which is present in Poynting's theorem, falls into the wave normal. We can easily convince ourselves, that it has the direction, in which the waves are propagating, and we find for its magnitude

$$4\pi a^2 (V + 2p_s) \cos^2 \psi'.$$

The energy flux through a plane which is parallel to the waves, thus amounts for the unit of area and time

§ 76. From a light bundle as the one considered above, others of the same kind can arise by refraction and mirroring at plane limiting surfaces. Here, we only consider such ones, that are again propagating in the aether, and we represent (for the case that the earth is at rest) one of the bundles, which emerge from the incident motion considered in § 74, by the following formula

§ 77. With this motion only that corresponds, which (in case Earth is moving together with the reflecting or refracting body) emerges from the light represented by (96)-(98). For this new state of motion we can thus write from which it again follows — see (100) and (101) —

In these equations l_1, m_1, n_1 determine the direction of the rays, which we also want to denote by s_1 .

§ 78. In the course of mirroring or refraction, the *absolute* period will be changed in general, while, as it nearly goes without saying and as it is also expressed by our formulas, the *relative* period is the same for all relevant light bundles. The absolute period of the incident motion is (§ 75)

$$T \left(1 - \frac{p_s}{V} \right).$$

Also, as regards the bundle considered in the previous paragraph it becomes

$$T \left(1 - \frac{p_{s1}}{V} \right).$$

Thus it has changed in the ratio of 1 to $1 + \frac{p_s - p_{s1}}{V}$.

If *e.g.* the rays are falling perpendicular upon a plate, which retreats by the velocity p in the direction of the perpendicular, then for the incident light $p_s = p$, and for the reflected light $p_{s1} = -p$. The variation of the absolute oscillation period during reflection will consequently be determined by the ratio $1 + \frac{2p}{V}$.

Also in the ratio between the amplitudes of the incident and the mirrored or refracted light, an influence of Earth's motion can be seen. The amplitude of the dielectric displacement \mathfrak{d} is namely with respect to the states of motion considered in §§ 74, 75, 76 and 77

$$a, a \left(1 + \frac{p_s}{V} \right), a_1, a_1 \left(1 + \frac{p_{s1}}{V} \right).$$

The ratio just mentioned is

$$\frac{a_1}{a},$$

in case the earth is at rest, and

$$\frac{a_1}{a} \left(1 + \frac{p_{s1} - p_s}{V} \right),$$

if it is moving.

In the case previously considered, where the rays are falling perpendicular to the retreating plate, the latter expression becomes

$$\frac{a_1}{a} \left(1 - \frac{2p}{V} \right);$$

the reflected light will thus be weakened by the motion of the plate. Of course, the opposite motion would strengthen it.

Now the important question emerges, whether these variations of intensity are in accordance with the conservation of energy. To decide this matter, we have to consider, that the aether (due to the motion of light), is acting by certain forces on the mirroring or refracting body (§ 17), and that these forces do some work, as soon as the body is displaced by the velocity p .

Now, we imagine (limited by plane surfaces and surrounded by aether) a transparent body K , upon which a system of plane waves is falling, and from which reflected and refracted light-bundles are emanating again. Let us put a *fixed*, closed surface σ around it, and calculate for a time interval which is equal to the *relative* period T ,

1°. the amount of energy A , which is flows rather in- than outwards through σ ,

2°. the growth B of the electric energy within the surface, and

3°. the work C of the forces mentioned above.

For simplification we assume, that the amplitudes be constant, and that the body will be continuously hit by rays in the same way, which is the case, when the light source, or the diaphragm that serves to limit a bundle of sunlight, shares the translation of K . After expiration of time T , the energy itself has again the original value within the body, and even the energy located in σ wouldn't be changed, when also the surface would be displaced by the velocity p . As regards the calculation of B , consequently only the energy in certain parts of space that lie in the direct vicinity of σ , come into consideration.

Eventually, we will find

by which it is proven, that we were always (as regards our developments) in agreement with the energy theorem.

However, I don't want to hinder myself with the verification of equation (104), since it might be preferable to treat the question more generally.

1.9 The conservation of energy in a more general case.

§ 79. An arbitrary transparent body K shall be hit by a homogeneous light motion, whose intensity remains constant; consequently, a certain motion arises in the body and in the aether in its vicinity.

Here, when Earth is at first imagined as stationary, the components of \mathfrak{d} and \mathfrak{H} in the aether are certain functions of x, y, z, t , and namely as regards the last variable, goniometric functions with the period T . During a complete period, *e.g.* in the time interval from $t_0 - T$ to t_0 , equal quantities of energy must flow in- and outwards through an arbitrary surface σ that surrounds the surface, which can be expressed by Poynting's theorem by

By assuming, that this condition is fulfilled, we want to show, that also state of motion that corresponds with that above, which can exist in the case of a translation p , satisfies the energy theorem.

If we replace in the functions, which apply to $\mathfrak{d}_x, \mathfrak{H}_x$, etc. when the Earth is at rest, the time t by the "local time" t' (§ 31), and if we understand in those functions by x, y, z the coordinates with respect to a movable system, then we obtain values of $\mathfrak{d}'_x, \mathfrak{H}'_x$, etc. for the new state. From (105) it thus directly follows, that

if it is presupposed, that we choose for σ a surface, which shares the motion of the body.

§ 80. However, now the flux of energy through a fixed surface σ shall now be considered. The energy flux related to its unit shall be

$$V^2[\mathfrak{p}.\mathfrak{H}]_n ,$$

or, as we find from the formulas (IX) and (VI_b) (§§ 56 and 20), under continuing omission of magnitudes of second order,

If we want, on that basis, to calculate the energy which flows more out- than inwards between the times $t_0 - T$ and t_0 , and consequently, by remarking the latter, integrate with respect to time. As regards the two latter terms, we can also think of a surface, that progresses with velocity \mathfrak{p} .

§ 81. To also arrange the integration of the first term in such a way, that we have to deal with such a movable surface, we at first set for the increase of the integral $V^2 \int [\mathfrak{p}.\mathfrak{H}]_n d\sigma$ at certain t , when we displace the surface σ in the direction of \mathfrak{p} about an infinitely small distance ϵ , the sign

$$\mathfrak{x} \in ,$$

where \mathfrak{x} is of course a very special function of t . Furthermore, we think of a surface σ_0 , which falls into σ at time t_0 , yet which is rigidly connected with earth. Then, at time t the "distance" of σ and σ_0 has the value $\mathfrak{p}(t_0 - t)$, which is to be considered as infinitely small, and our integral for the fixed surface σ amounts

$$\mathfrak{p}\mathfrak{x}(t_0 - t) ,$$

more than for σ_0 . The time integral, about which we speak eventually, is thus about

greater than the time integral taken for σ_0 , and, since the latter vanishes by (106), we have only to deal with the value (108).

By the way, in \mathfrak{x} we don't have to consider the magnitudes containing \mathfrak{p} , and thus we may understand, since with this omission

$$V^2 \int [\mathfrak{p}.\mathfrak{H}]_n d\sigma ,$$

is the energy flux, under

$$\mathfrak{x} \in$$

the difference calculated for the unit time, and under

$$\mathfrak{x} \in dt$$

the difference calculated for the element dt , of the energy fluxes through two fixed surfaces that are mutually distant by the length ϵ .

Now, let $Q\epsilon$ be the energy, which at time t is more surrounded by our surface σ in its fixed location, as when this surface would be displaced by ϵ in the direction of \mathfrak{p} ; then we immediately see, that

By that, and furthermore by partial integration, (108) is transformed into

$$\mathfrak{p} \int_{t_0-T}^{t_0} \frac{dQ}{dt} (t_0 - t) dt = -\mathfrak{p} T Q_{t=t_0-T} + \mathfrak{p} \int_{t_0-T}^{t_0} Q dt ,$$

or

$$-\mathfrak{p} T Q_{t=t_0} + \mathfrak{p} \int_{t_0-T}^{t_0} Q dt ,$$

since, except magnitudes of order \mathfrak{p} , Q has again the original value after the expiration of time T .

§ 82. Until now, we only spoke about the first member in (107). If we denote the two other members by A , then we have

$$-\mathfrak{p} T Q_{t=t_0} + \mathfrak{p} \int_{t_0-T}^{t_0} Q dt + \int_{t_0-T}^{t_0} dt \int A d\sigma$$

the complete value of the energy, that travelled outwards through σ . If we add the increase of the energy in the interior of σ , and the work of the forces, by which the aether is acting on the ponderable body, then we must, if the energy theorem shall be satisfied, obviously obtain zero.

The increase energy in a full period T would be zero, if the surface σ together with the body K would be displaced over the distance $\mathfrak{p}T$, and at this place would have taken the location σ'' ; it factually consists of the energy amount, which, at time t_0 , is more contained in σ than in σ'' . This is now, as it follows from the definition given for $Q\epsilon$, exactly

$$\mathfrak{p} T Q_{t=t_0} .$$

The work mentioned above can be expressed, as we will see soon, by an expression of the form

$$\int_{t_0-T}^{t_0} dt \int S d\sigma ;$$

thus the energy law requires that

$$\mathfrak{p} \int_{t_0-T}^{t_0} Q dt + \int_{t_0-T}^{t_0} dt \int A d\sigma + \int_{t_0-T}^{t_0} dt \int S d\sigma = 0$$

If it is additionally achieved, to represent Q as an integral over σ , e.g. in the form

$$Q = \int q d\sigma$$

and to show, that

then we have achieved our goal.

§ 83. From the definition given for $Q\epsilon$ we derive, that by $q\epsilon d\sigma$ we have to understand the energy content of the space, which is traversed by the element during the displacement ϵ , and namely we have, depending on whether the displacement takes place with respect to the inner- or the outer-side of σ , to apply the positive or the negative sign. Thus we have

$$q \epsilon d\sigma = -\epsilon \cos(\mathbf{p}, \mathbf{n}) \left(2\pi V^2 \mathfrak{d}^2 + \frac{1}{8\pi} \mathfrak{H}^2 \right) d\sigma ,$$

and

$$\mathbf{p}q = -\mathbf{p}_n \left(2\pi V^2 \mathfrak{d}^2 + \frac{1}{8\pi} \mathfrak{H}^2 \right) .$$

Second, as regards the work, we don't have to care about the last member in equation (15) and the analogous formulas.^[21] Only the “tensions” come into account, and

$$S \, d\sigma \, dt$$

is the work of the tension with respect to $d\sigma$. The components of this tension are

$$\left\{ 2\pi V^2 (2\mathfrak{d}_x \mathfrak{d} - \alpha \mathfrak{d}^2) + \frac{1}{8\pi} (2\mathfrak{H}_x \mathfrak{H}_n - \alpha \mathfrak{H}^2) \right\} d\sigma , \text{ etc.},$$

from which it follows

Eventually, A means the sum of the two last members in (107).

The given values now actually satisfy the condition (109).

[1] That this is also the case during the observation by a mirror telescope, would also follow from our theorem, *when the mirror would consist of transparent material*. However, as regards the actual mirrors that are constructed by metal, we can remark, that the direction by which the light rays are reflected, and the *location of the unification point* only depends on the curvature, but not on the material nature of the mirror. For the determination of this location, as it was done by various physicists, also the Principle of Huygens can be applied, (see also my treatise in Arch. neerl., T. 21).

[2] Arago. OEuvres complètes, T. 1, p. 107; Biot. Traité élémentaire d'astronomie physique, 3e éd., T. 5, p. 364.

[3] Airy. Proc. Royal Society of London, Vol. 20, p. 35, 1871; Vol. 21, p. 121, 1873; Phil. Mag., 4th Ser., Vol. 43, p. 310, 1872.

[4] We neglect the rotation of the sun and the motions at its surface, from which it is known that they cause a displacement of the spectral lines in accordance with Doppler's law. As regards the experiments that will mentioned soon, light of the *whole disc of the sun* was used.

[5] Mascart. Ann. de l'école normale, 2e ser., T. 1, pp. 166—170, and p. 190, 1872.

[6] Mascart. L. c., pp. 170 and 189.

[7] During the experiments with sun-light, of course, metallic mirrors were used. However, we can easily see, that this changes nothing as regards our considerations (see the note 1 at p. 89)

[8] Respighi. Memor. di Bologna (2), II, p. 279. (Cited in Ketteler. Astronomische Undulationstheorie, p. 66).

[9] Hoek. Astr. Nachr., Bd. 73, p. 193.

[10] Ketteler. Astr. Und.-Theorie, p. 66, 1873; Pogg. Ann., Bd. 144, p. 370, 1872.

[11] Mascart. Ann. de l'école normale, 2e sér., T. 3, p. 376, 1874.

[12] Hoek. Arch. neerl., T. 3, p. 180, 1868. Ketteler. Astr. Und.-Theorie, p. 67; Pogg. Ann., Bd. 144, p. 372. Mascart. L. c., pp. 390—416.

[13] Ketteler. Astr. Und.-Theorie, pp. 158 and 166; Pogg. Ann., Bd. 147, pp. 410 and 419, 1872.

[14] Mascart. Ann. de l'école normale, 2e sér., T. 1, pp. 191—196, 1873

[15] A derivation of equation (84) from the electromagnetic light theory was published by R. Reiff Wied. Arm., Bd. 50, p. 861, 1893). Long before me, also J. J. Thomson has dealt with this subject (Phil. Mag., 5th. Ser., Vol. 9, p. 284, 1880; Recent Researches in Electricity and Magnetism, p. 543), however, without obtaining Fresnel's coefficient.

[16] Michelson and Morley. American Journal of Science, 3d Ser., Vol. 81, p. 377, 1886.

[17] In the following formulas of this paragraph, p simply means the *magnitude* of velocity.

[18] Veltmann. Pogg. Ann., Bd. 150, p. 497, 1873.

[19] Lorentz. Arch. néerl., T. 21.

[20] A derivation of this theorem from formula (87) was published by me in Zittingsverslagen of the Akad. T. Wet. te Amsterdam, 1892—93, p. 149,

[21] Namely, to calculate the work, we can multiply the path pT with the average of the force acting in its direction. This average would be zero for the last member in (15), when the surface σ would be displaced together with the body, from which it follows, that it is in reality of order p .

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2.1 Text

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